

# Precise Orbit Determination of Low Earth Satellites at AIUB using GPS and SLR

A. Jäggi<sup>1</sup>, H. Bock<sup>1</sup>, D. Thaller<sup>2</sup>, K. Sošnica<sup>1</sup>,  
U. Meyer<sup>1</sup>, C. Baumann<sup>1</sup>, R. Dach<sup>1</sup><sup>1</sup> Astronomical Institute, University of Bern,  
Bern, Switzerland<sup>2</sup> German Federal Agency of Cartography and  
Geodesy, Frankfurt a.M., Germany

## Introduction

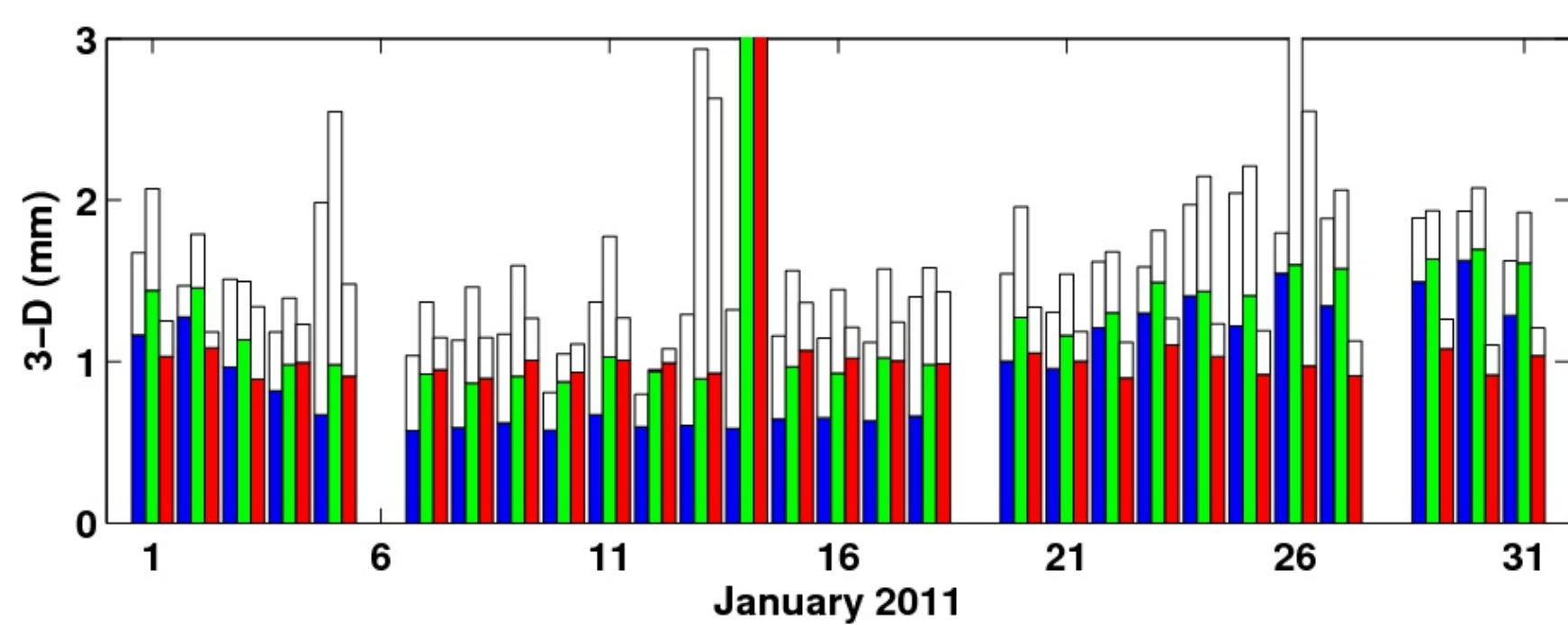
The Astronomical Institute of the University of Bern (AIUB) has a well-documented record concerning the scientific analysis of Global Navigation Satellite System (GNSS) data with the Bernese GNSS Software. The Center for Orbit Determination in Europe (CODE), a global analysis center of the International GNSS Service (IGS) located at AIUB, generates the full IGS product line, in particular GNSS orbits and high-rate satellite clock corrections, which are a prerequisite for spaceborne GPS applications. Precise kinematic and reduced-dynamic orbits relying on the CODE products are generated at AIUB for a variety of Low Earth Orbiting (LEO) satellites. So-called pseudo-stochastic orbit modeling techniques have been continuously developed and refined and allow for efficient and flexible LEO precise orbit determination (POD) from spaceborne GPS data. The procedures are operationally used by AIUB to derive the Precise Science Orbits (PSO) in the frame of the GOCE High-level Processing Facility (HPF).

The Bernese GNSS Software has recently been extended from a pure GNSS processing software to a package also offering full capabilities for processing Satellite Laser Ranging (SLR) data. Identical orbit modeling techniques as used for GPS-based LEO POD are applied when processing SLR data to solve for orbital parameters together with SLR station coordinates, Earth rotation parameters, SLR-specific range biases, and geopotential coefficients.

## TerraSAR-X / TanDEM-X



AIUB initiated an inter-agency comparison of precise baseline solutions of the TanDEM-X mission generated at AIUB, the German Research Centre for Geosciences (GFZ), and the German Space Operations Center (DLR). Reduced-dynamic baseline vectors based on double-difference dual- or single-frequency GPS data and fixed carrier phase ambiguities were compared with each other. The parameters of the reduced-dynamic baseline solutions are the six initial osculating elements, three constant empirical accelerations acting over the 24h arc in the radial, along-track, and cross-track directions, and piecewise constant accelerations over 6 min acting in the same directions. The entire sets of empirical accelerations of both formation-flying satellites are tightly constrained with respect to each other. Frequently performed maneuvers are modeled by instantaneous velocity changes. The agreement of the single-frequency solutions of the three agencies outside maneuver periods is at standard deviations of approximately 0.9 mm (Jäggi et al., 2012).

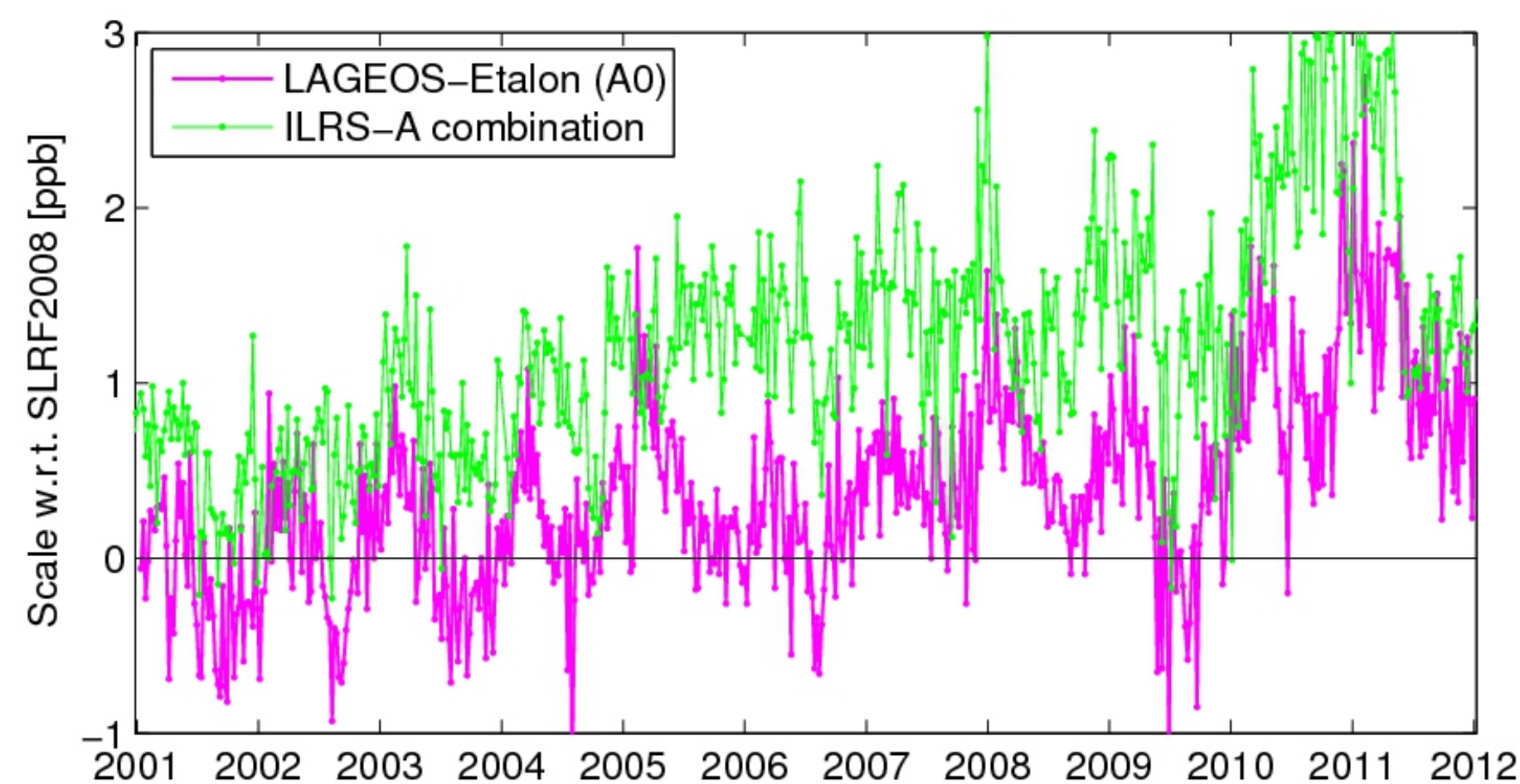


**Figure:** Daily standard deviations of inter-agency single-frequency baseline comparisons. Empty bars indicate the statistics for entire 24h arcs, colored bars exclude maneuver periods (in blue: GFZ-DLR, in green: GFZ-AIUB, in red: AIUB-DLR).

## LAGEOS-1/2

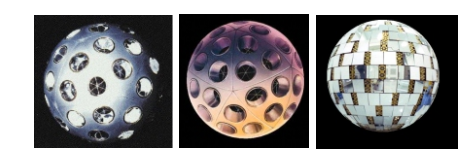


LAGEOS satellites are orbiting the Earth at an altitude of almost 6000 km. Therefore, and due to the generally sparse SLR tracking coverage, a purely dynamic orbit representation is aimed at. The strongest non-gravitational perturbations caused by solar radiation pressure may be well described for spherical satellites and are taken into account by a priori models. LAGEOS 7-day arcs are thus generated with only a few additional empirical orbit parameters. Apart from the six initial osculating elements, one constant acceleration in the along-track direction and once-per-rev accelerations in the along-track and cross-track directions acting over the entire 7-day arcs are set up (Thaller et al., 2013). Weekly solutions for the LAGEOS (and Etalon) satellite orbits are estimated at AIUB together with Earth rotation parameters and station coordinates following the standards of the analysis centers of the International Laser Ranging Service (ILRS).



**Figure:** Scale differences w.r.t. SLRF2008 for the LAGEOS-Etalon solutions and the official ILRS combined solutions. The agreement with SLRF2008 is even better for AIUB than for the routinely generated official ILRS-A combined solutions, because the set of stations used for the datum definition is verified for each weekly AIUB solution.

## Starlette / Stella / AJISAI

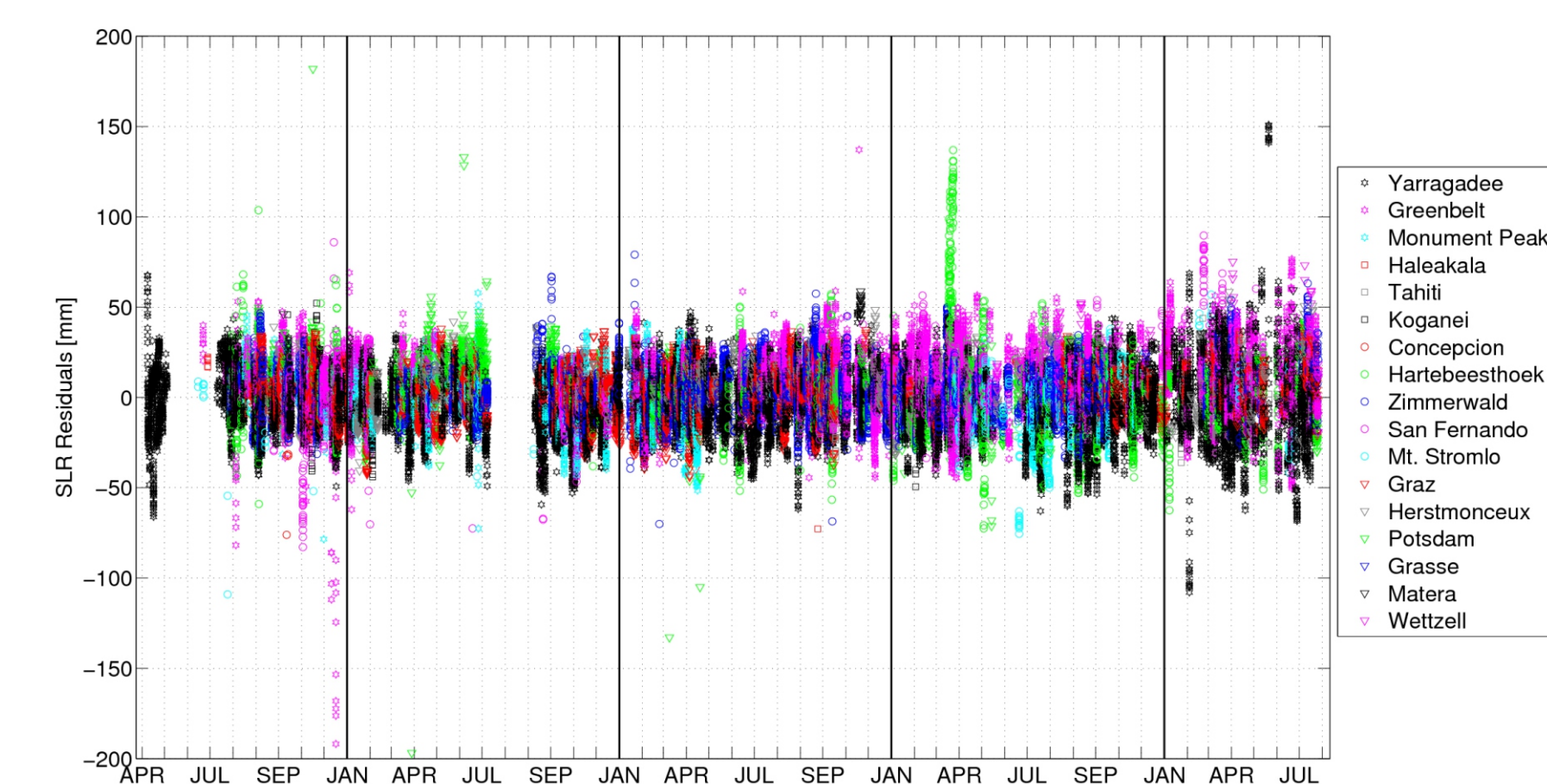


Starlette, Stella, and AJISAI 7-day solutions are computed in close analogy to LAGEOS 7-day solutions with range biases estimated for all SLR stations, because of a lack of precise Center-of-Mass corrections for these satellites. Due to the much lower orbital altitudes the orbit parametrization has to account for air-drag as a relevant non-gravitational perturbation, which is modeled according to the NRLMSISE-00 model. Scaling factors and once-per-rev accelerations are estimated in the along-track ...

## GOCE



AIUB is responsible for the generation of the PSO product of the GOCE mission (Bock et al., 2011). The parameters of the reduced-dynamic orbit of the PSO product are the six initial osculating elements, three constant empirical accelerations acting over the entire 30h arc in the radial, along-track, and cross-track directions, and piecewise constant accelerations over 6 min acting in the same directions. No use is made of the GOCE common-mode accelerometer data for the official solution. Due to the low orbital altitude only weak constraints are imposed on the piecewise constant accelerations. The quality of the reduced-dynamic orbits is externally validated by independent SLR data at an RMS of about 1.7 cm.



**Figure:** SLR residuals from April 10, 2009 - July 31, 2013 for the reduced-dynamic GOCE PSO solutions.

## Fundamentals of Orbit Determination

The equation of motion of an Earth orbiting satellite including all perturbations reads in the inertial frame as

$$\ddot{\mathbf{r}} = -GM \frac{\mathbf{r}}{r^3} + \mathbf{f}_1(t, \mathbf{r}, \dot{\mathbf{r}}, Q_1, \dots, Q_d, P_1, \dots, P_s)$$

with  $\mathbf{r}(t_0) = \mathbf{r}(a, e, i, \Omega, \omega, u_0; t_0)$  and  $\dot{\mathbf{r}}(t_0) = \dot{\mathbf{r}}(a, e, i, \Omega, \omega, u_0; t_0)$  being initial conditions defined by six Keplerian osculating elements. The parameters  $Q_1, \dots, Q_d$  denote additional dynamical orbit parameters considered as unknowns, e.g., coefficients of the Earth's gravity field. The parameters  $P_1, \dots, P_s$  denote additional empirical parameters, e.g., once-per-rev accelerations or pseudo-stochastic parameters.

Based on a numerically integrated a priori orbit, orbit determination may be formulated as an orbit improvement process. The actual orbit is expressed as a truncated Taylor series with respect to all unknown orbit parameters about the a priori orbit:

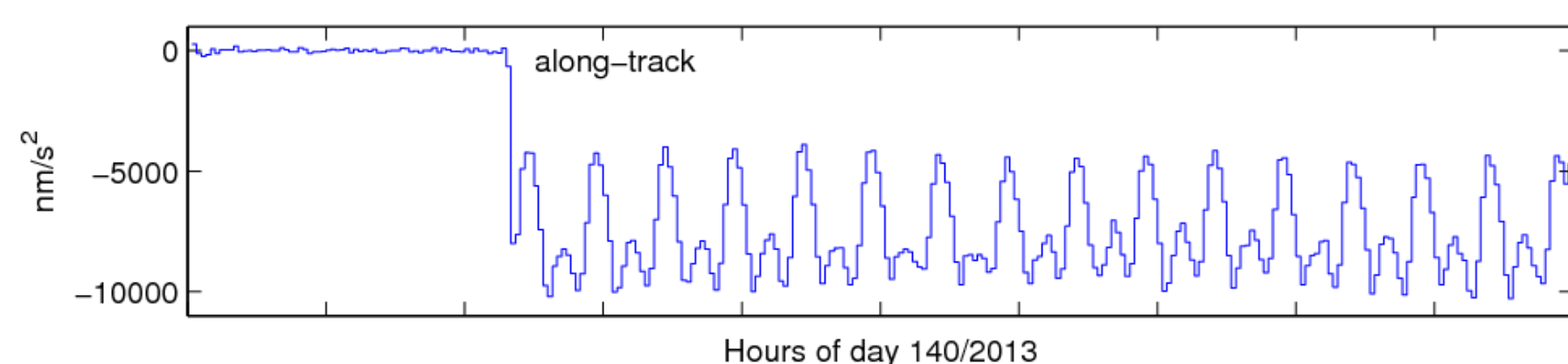
$$\mathbf{r}(t) = \mathbf{r}_0(t) + \sum_{i=1}^n \frac{\partial \mathbf{r}_0}{\partial P_i}(t) \cdot (P_i - P_{0,i})$$

Corrections to the a priori orbit parameters are estimated in a standard least-squares adjustment of the tracking data together with measurement-specific and other non-orbit parameters.

The needed partials are obtained from the system of variational equations. For each parameter  $P_i$  the system has the form:

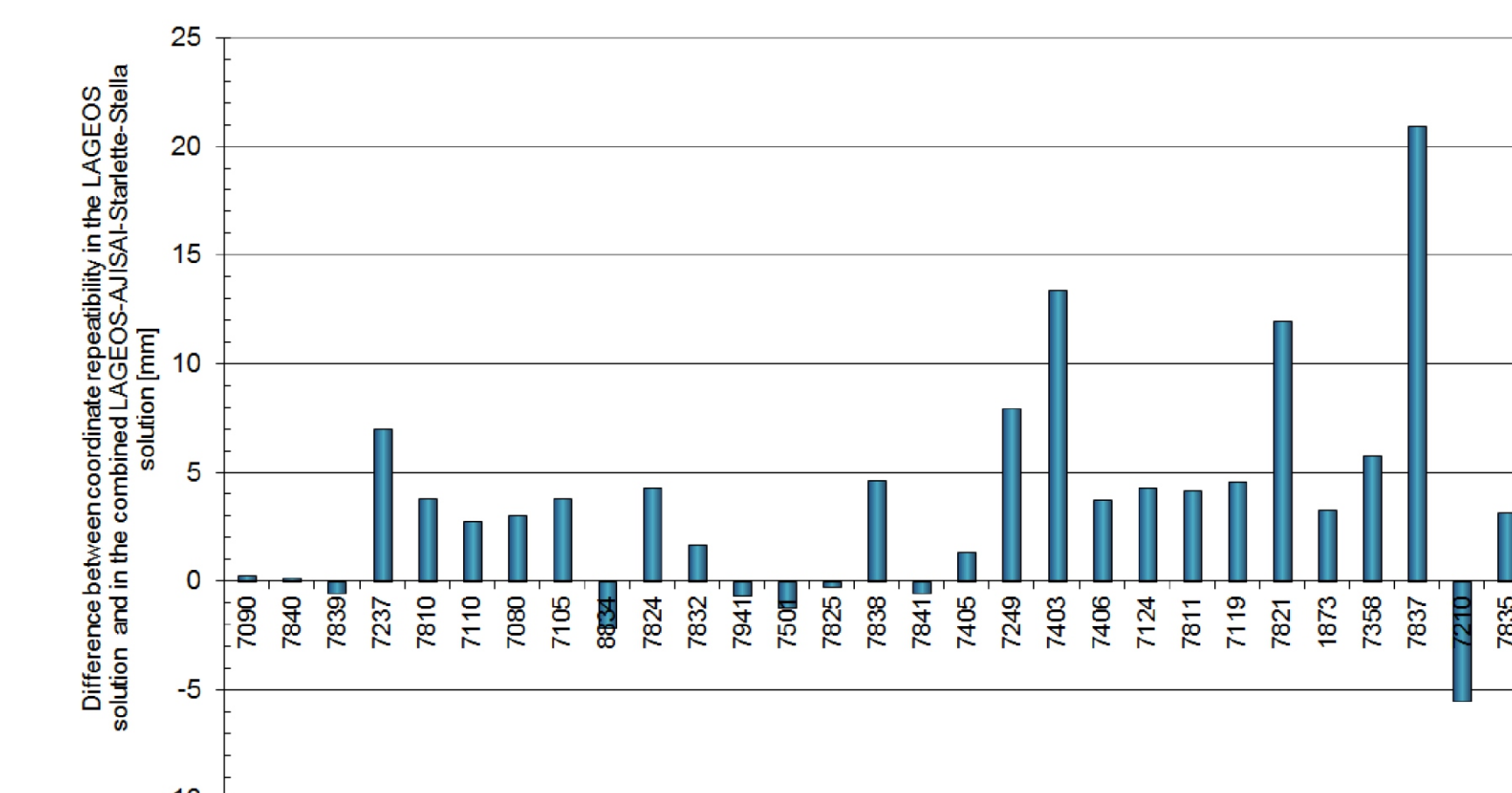
$$\ddot{\mathbf{z}}_{P_i} = \mathbf{A}_0 \cdot \mathbf{z}_{P_i} + \mathbf{A}_1 \cdot \dot{\mathbf{z}}_{P_i} + \frac{\partial \mathbf{f}_1}{\partial P_i}$$

Pseudo-stochastic parameters are primarily intended to compensate for force model deficiencies and are characterized by a priori variances which constrain them to zero. If dense tracking data are available, e.g., GPS data, pseudo-stochastic parameters such as piecewise constant accelerations may be set up frequently and can even be used to replace deterministic force models, e.g., models for atmospheric drag (Jäggi et al., 2006).



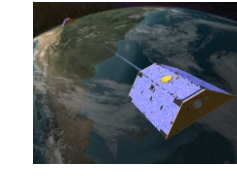
**Figure:** Piecewise constant accelerations estimated for GOCE on 20 May, 2013 at the altitude of 234 km. Note the transition into non-drag-free flight to further lower the orbit.

... and cross-track directions on a daily basis. Pseudo-stochastic pulses are estimated every revolution period in the along-track direction with an a priori standard deviation of  $10^{-7}$  m/s. The combination with weekly LAGEOS solutions is beneficial for estimating satellite orbits together with Earth rotation parameters and station coordinates. The a priori sigmas of the individual contributions are set to 25, 20, 20 and 10mm for AJISAI, Starlette, Stella, and LAGEOS, respectively (Sošnica et al., 2013). Station coordinates, e.g., show a better repeatability when combining the LAGEOS solutions with the solutions for Starlette, Stella, and AJISAI.

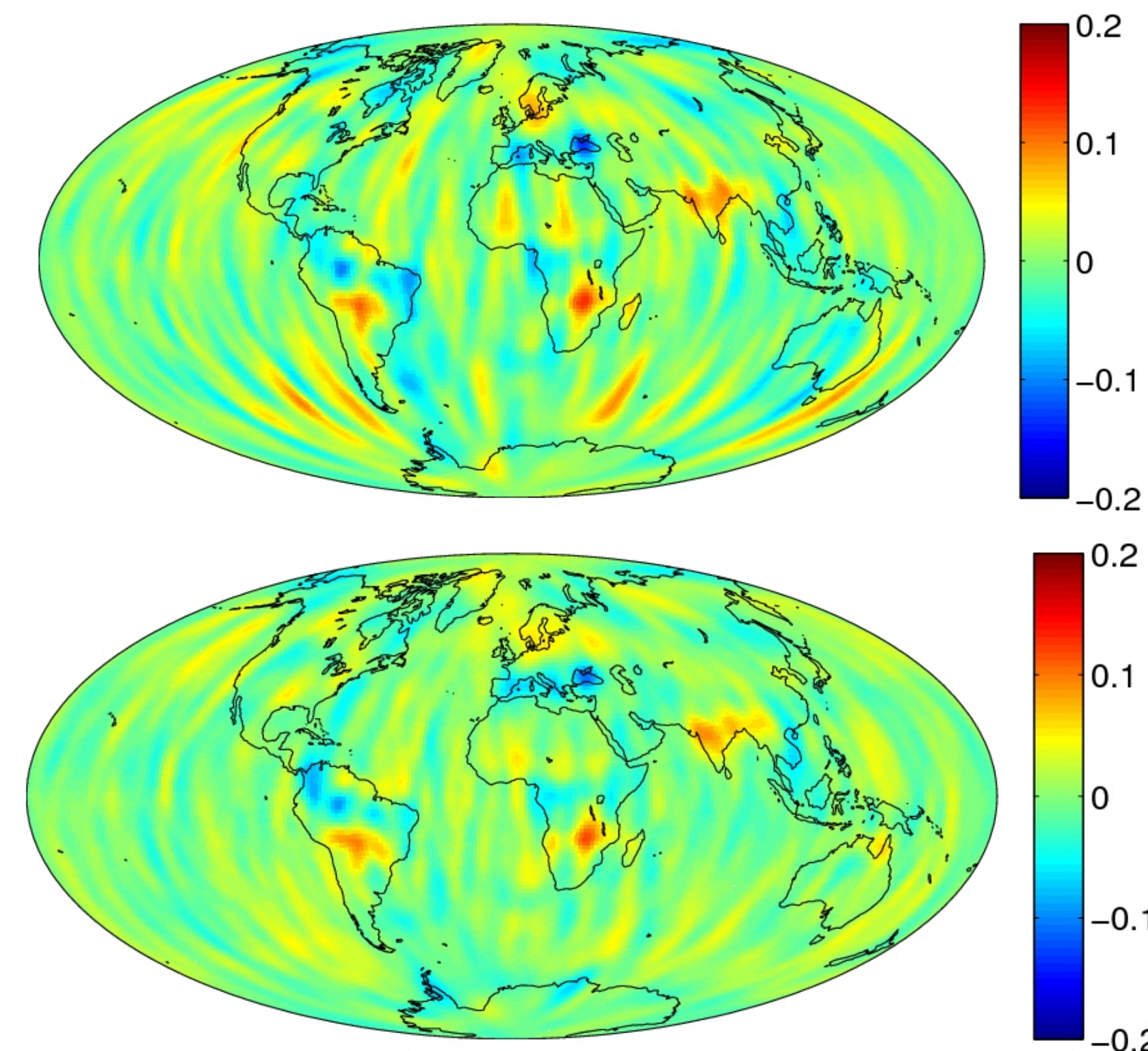


**Figure:** Difference of the repeatability of station coordinates in the combined solution and in the LAGEOS-1/2 solution. Positive values denote a better repeatability in the combined solution.

## GRACE



Monthly gravity field determination from ultra-precise inter-satellite K-Band data and kinematic GRACE positions is performed at AIUB by simultaneously estimating gravity field coefficients and arc-specific orbit parameters in the frame of a generalized orbit determination problem (Meyer et al., 2012). Separating the estimation of arc-specific orbit parameters and global gravity field parameters, e.g., by fixing orbit parameters to estimates obtained when adopting a good a priori gravity field model, considerably reduces the noise (striping) of the monthly gravity field solutions. The consequences for the signal content are currently under investigation.

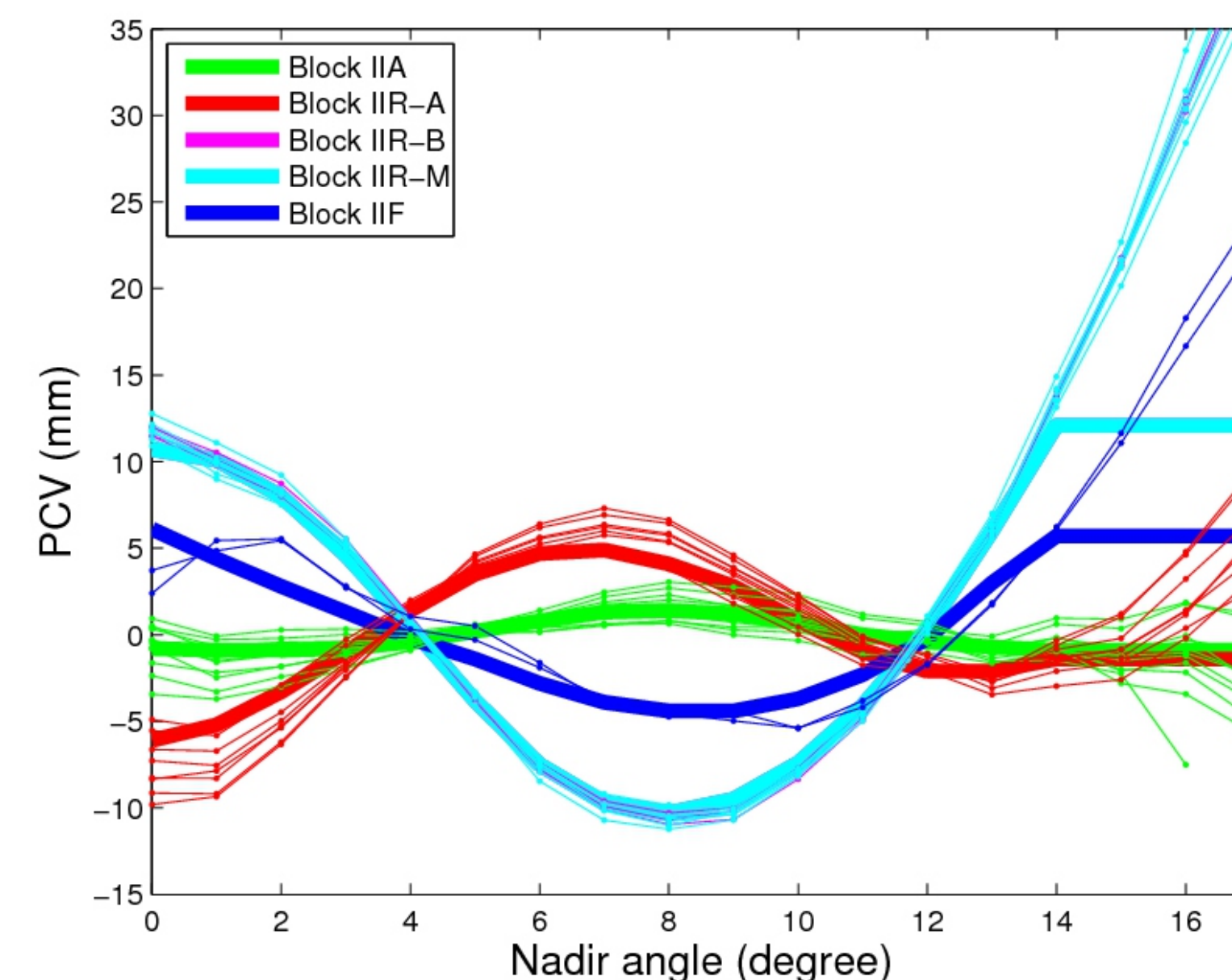


**Figure:** Difference of the monthly gravity field of March 2008 up to degree 60 with respect to the time variable signal of AIUB-GRACE03S (only modeled by annual, semi-annual, and trend signals) when simultaneously (top) or separately (bottom) solving for orbit and gravity field parameters. The scale corresponds to  $\pm 20$ cm water height.

## Jason-2 / MetOp-A



Until recently the absolute phase center model for GNSS transmitter antennas was solely based on terrestrial GNSS data, which limited the estimation of GPS and GLONASS satellite antenna PCVs to a maximum nadir angle of  $14^\circ$ . This is not sufficient for the analysis of spaceborne GNSS data collected by LEO satellites that record – depending on the missions' orbital altitude – observations at nadir angles of up to  $17^\circ$ . GPS tracking data from several LEO missions, e.g., Jason-2 and MetOp-A, were used to extend the GPS satellite antenna PCVs to nadir angles beyond  $14^\circ$ . Satellite-specific GPS and LEO antenna PCVs were simultaneously estimated to derive the recently published block-specific extension of the igs08.atx model beyond nadir angles of  $14^\circ$  (Schmid et al., 2013).



**Figure:** Estimated satellite-specific PCVs using LEO data (thin lines) and igs08.atx block-specific values before the PCV extension beyond nadir angles of  $14^\circ$  (bold lines).

## Conclusions

LEO POD using either GPS or SLR data is performed at AIUB for satellites at very different altitudes and for a wide range of applications. Provided that dense GPS tracking data is available, accuracies of better than 2cm are achieved by pseudo-stochastic orbit modeling for LEO satellites at low orbital altitudes such as GOCE. Further extensions of the Bernese GNSS Software will include combined orbit determination from GPS and SLR data.

## References

- Bock H., Jäggi A., Meyer U., Visser P., van den Ijssel J., van Helleputte T., Heinze M., Hugentobler U., 2011, GPS-derived orbits for the GOCE satellite. *Journal of Geodesy*, 85(11), 807-818
- Jäggi A., Hugentobler U., Beutler G., 2006, Pseudo-stochastic orbit modeling techniques for Low-Earth orbiters. *Journal of Geodesy*, 80(1), 47-60
- Jäggi A., Montenbruck O., Moon Y., Wermuth M., König R., Michalak G., Bock H., Bodenmann D., 2012, Inter-agency comparison of TanDEM-X baseline solutions. *Advances in Space Research*, 50(2), 260-271
- Meyer U., Jäggi A., Beutler G., 2012, Monthly gravity field solutions based on GRACE observations generated with the Celestial Mechanics Approach. *Earth and Planetary Science Letters*, 345-348, 72-80
- Schmid R., Dach R., Collilieux X., Jäggi A., Schmitz M., Dißner F., 2013, Absolute IGS antenna phase center model igs08.atx: status and improvements. *Journal of Geodesy*, in preparation
- Sošnica K., Jäggi A., Thaller D., Dach R., Beutler G., 2013, Contribution of Starlette, Stella, and AJISAI to the SLR-derived global reference frame. *Journal of Geodesy*, in preparation
- Thaller D., Sošnica K., Beutler G., Jäggi A., Dach R., Mareyen M., 2013, Geodetic parameters estimated from eleven years of LAGEOS and Etalon data. *Journal of Geodesy*, in review

## Contact address

Adrian Jäggi  
Astronomical Institute, University of Bern  
Sidlerstrasse 5  
3012 Bern (Switzerland)  
adrian.jaeggi@aiub.unibe.ch

